

IDEA SPACE

A Mathematical Model for Observing the Mind

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“In order to understand physical laws you must understand that they are all some kind of approximation.”

— *Richard Feynman*

Abstract

This paper proposes an **idea space** as a mathematical model for observing the mind. The main elements of an idea space are thoughts, emotions, sensations, and perceptions, together with the possibility of an empty category or state. It has three proposed properties. First, it is unique to the person living through it. Second, it has zero measure under a specified outside-observer measure: through that measurement, it receives the same value as the empty set even though it contains experience for the person within it. Third, its complete field of possible states is uncountable, meaning it cannot be completely listed. Lastly, the paper explores the Sunset Conjecture to show how an idea space fits within a person’s observable universe.

1 Introduction

The goal of an idea space is to address the following questions: How does the mind relate to the universe at large? Is there a way to structure the mind in a mathematical format based on observations like those used in modern physics?

Understanding how our mind relates to the universe has been one of life’s biggest questions since the dawn of time. Philosophers and mathematicians such as Michel de Montaigne, Blaise Pascal, Immanuel Kant, Bertrand Russell, and Albert Einstein approached parts of this mystery in their own ways. Today, the problem is no less prominent. Our model of self shapes suffering, identity, comparison, and responsibility, yet the boundary between a person and the surrounding universe is difficult to locate precisely.

An idea space provides a way to look at thoughts, emotions, sensations, and perceptions as objectively as one would view objects in spacetime—devoid of “I.” It does not claim to reveal the final nature of consciousness. It offers an approximate mathematical model for observing what appears in the mind. In this limited sense, an idea space continues the observer-relative lesson of relativity by looking more deeply at the uniqueness of lived experience.

1.1 What is physics?

Our understanding of physics does not define a perfect copy of reality. Specifically, physics uses mathematical models to describe nature as accurately as observation allows. Richard Feynman

captures the difficulty of even defining an ordinary chair:[11]

“Any simple idea is approximate; as an illustration, consider an object, ... what is an object? Philosophers are always saying, ‘Well, just take a chair for example.’ The moment they say that, you know that they do not know what they are talking about any more. What is a chair? Well, a chair is a certain thing over there ... certain?, how certain? The atoms are evaporating from it from time to time—not many atoms, but a few—dirt falls on it and gets dissolved in the paint; so to define a chair precisely, to say exactly which atoms are chair, and which atoms are air, or which atoms are dirt, or which atoms are paint that belongs to the chair is impossible. So the mass of a chair can be defined only approximately. In the same way, to define the mass of a single object is impossible, because there are not any single, left-alone objects in the world—every object is a mixture of a lot of things, so we can deal with it only as a series of approximations and idealizations.”

Mathematics establishes truths that follow from rigorous definitions and axioms. Physics takes those mathematical structures and applies them to observations of our world. The mathematics may be exact while the physical model remains provisional.

The atom makes the point. It has been pictured as a solid sphere, reorganized around Rutherford’s nucleus, drawn in the Bohr model with electrons following fixed, quantized orbits, described in quantum mechanics through orbitals or probability “clouds” with no classical electron trajectory, and treated in modern quantum field theory through fields and their excitations. Each model organizes a range of observations, and later models clarify where earlier approximations work and where they do not. Feynman makes a related point through the rough empirical law for the drag on an airplane:[11]

“If we continue to study it more and more, measuring more and more accurately, the law will continue to become more complicated, not less. In other words, as we study this law of the drag on an airplane more and more closely, we find out that it is ‘falsier’ and ‘falsier,’ and the more deeply we study it, and the more accurately we measure, the more complicated the truth becomes.”

His example concerns a semi-empirical drag law rather than every law of physics. It nevertheless shows how an effective mathematical description can remain useful while increasingly precise observations reveal the limits of its domain.

As a final example, Newton used calculus to translate observations of planetary motion into a mathematical model of gravity. The model predicted so much, so well, for so long that it became known as Newton’s law of universal gravitation. Yet it did not fully account for Mercury’s anomalous perihelion precession; general relativity supplied a broader model.[9, 6] Newton’s calculus did not become useless. Its extraordinary accuracy within a certain range became clearer, as did the point at which a new approximation was needed. Physics is therefore not the final, all-encompassing truth of nature. It is an immensely powerful and useful approximation of nature.

What makes these approximations scientific is falsifiability. We can seldom prove a physical theory true for all time. Instead, science proceeds through what Karl Popper called “conjectures and refutations”—bold explanations exposed to observations capable of showing where they fail. Popper writes:[23]

“Scientific theories [are] not the digest of observations, but they are inventions—conjectures boldly put forward for trial, to be eliminated if they clashed with observations; with observations which [are] rarely accidental but as a rule undertaken with the definite intention of testing a theory by obtaining, if possible, a decisive refutation.”

A conjecture proposes an explanation; a refutation is an observation the explanation cannot survive without revision. Elementary statistics uses similar language: after testing a hypothesis, we either reject it or fail to reject it. We do not prove it permanently true. Repeated success builds confidence, and a model may eventually be treated as a natural law within its tested range. Newtonian gravity shows both sides of that process: it remains a powerful law within its domain, while Mercury exposed a limit that a broader theory had to explain. This cycle raises the next question: if different domains require different observations and instruments, what kind of science can study the mind from the first person?

1.2 The science of objects and the science of the first person

One of the key properties of an idea space is that it has zero measure under the outside-observer measure developed below. In simple terms, zero measure looks like nothing. The later sections make this intuition precise and explain the limits of that comparison. Therefore, studying idea spaces requires new instrumentation.

The **science of objects** studies observations that can be measured through traditional instruments. Rulers, clocks, cameras, sensors, and experiments allow different observers to measure a shared object. Physics, chemistry, biology, geology, and the other natural sciences depend on this kind of access.

The **science of the first person** studies experience as it is directly presented to the person having it. In the model, an idea space has zero measure from an outside observer’s perspective. Activity in the brain and body can be measured from outside, while another person does not receive the subject’s direct, first-person experience. Figure 1 shows the distinction between the science of objects, which relies on shared measurement, and the science of the first person, in which another person’s experience cannot be measured directly by an outside observer.

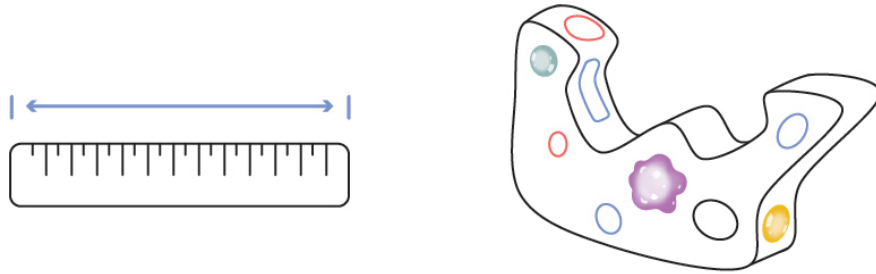


Figure 1: The science of objects and the science of the first person. (Left) Natural sciences can be measured through shared instruments. (Right) Private experience, like the idea space shown, is directly observed by the person having it and cannot be directly measured by others.

Because traditional instruments cannot directly measure another person’s first-person experience, we

need another observational tool. By applying the same principles of falsifiability and reproducibility that have made traditional science successful, we can develop a science of the first person. Anagarika Munindra offered one plain instruction:[12]

“If you want to understand your mind, sit down and observe it.”

The main tool we have at our disposal is mindfulness. Jon Kabat-Zinn gives a working definition of the method:[15]

“Mindfulness is the awareness that arises from paying attention, on purpose, in the present moment, and non-judgmentally.”

Mindfulness is disciplined first-person observation, not an infallible source of truth. Memory, expectation, attention, interpretation, and reporting can all introduce error. Its observations can nevertheless be repeated, compared, challenged, and related to behavior, physiology, and neural activity.[26] The sciences of objects and of the first person are complements, not rivals. The key is giving first-person observation a structured and explicit language. This is one of the main goals of an idea space: to provide a common framework for describing and studying the mind.

A rigorous science of first-person experience requires reproducible methods and hypotheses exposed to possible refutation. The three key hypotheses of an idea space can be stated alongside their present observations and mathematical forms:

- **Unique to the individual.** *Present observation and test:* no two people occupy the same brain, experiential history, or first-person perspective. Experiences can be reported and compared, but each presentation remains situated in the individual having it. *Mathematical form:* contents and states are indexed by their subject, so distinct people have distinct personal idea spaces.
- **Zero external measure.** *Present observation and test:* an outside observer can study reports, behavior, physiology, and neural activity, but cannot directly see or occupy another person’s first-person experience. *Mathematical form:* the chosen constant-zero pseudometric assigns zero distance between all states from the outside-observer perspective and consequently yields zero external measure.
- **Uncountable possibility and impermanence.** *Present observation and test:* lived experience changes from moment to moment, while sensory and mental distinctions admit continued refinement. *Mathematical form:* momentary states represent ongoing change, while taking the full power set of a countably infinite ingredient set produces an uncountable field of possible states.

These are not declarations of final truth. They are a mathematical description of the mind based on the information available today. Brain-computer interfaces, including recent Neuralink demonstrations, can already decode neural activity associated with attempted speech into text or synthesized speech.[28, 21] This does not yet transfer the full first-person experience itself, but it’s a step in the right direction. Future advances could refute the zero-external-measure hypothesis, and that openness to refutation is why science is so beautiful.

For now, these hypotheses are retained while they withstand testing and remain open to revision if future observations refute them—just as Newtonian gravity remained useful within its domain even after a broader model became necessary.

1.3 The Sunset Conjecture

The Sunset Conjecture is the bridge between the science of the first person and the science of objects. It is best illustrated through a short story.

Picture yourself on the beach. You had a fantastic day of doing nothing but reading, hanging out, and talking with friends. At the end of the day, you decide to walk on the beach to watch the sunset. You say to your friends, “I must be special. It seems the sun’s golden rays are reflecting right off the water directly toward me.” Your friend responds, “No, you idiot. The golden rays are pointing directly toward me.” Clearly, you are both right. Figure 2 illustrates the shared scene.



Figure 2: The Sunset Conjecture. The same sunset sends different reflected paths toward two observers standing in different positions.

This phenomenon can best be explained by the two-fold Sunset Conjecture:

- (a) every observer lives at the center of their own observable universe where everything they see is in the past; and
- (b) their idea space, with uncountable possibility and zero external measure, is associated with the observer at that central observer-event.

The first statement does not place anyone at a privileged physical center of the entire universe. An observable universe is defined relative to an observer.

Everyone shares one physical universe, yet everyone experiences a different perspective of it. Pick any object in the universe and its relation to you depends on where and when you encounter it. Near the end of the paper, we extend this idea toward the edge of the observable universe and the early universe itself. This is the Singularity Sunset: not a separate creation for each person, but a unique perspective on a shared cosmic history.

The path ahead defines the elements of an idea space, distinguishes the empty set from zero measure, develops its pseudometric and topology, and uses the Cantor set to make its uncountability precise before returning to the sunset.

2 The elements of an idea space

Drawing inspiration from Zen and first-person observation, the practical description of an idea space has four main elements—thoughts, emotions, sensations, and perceptions—as well as the possibility of emptiness.

1. **Thoughts** include words, music, images, daydreams, plans, memories, and general mental formations.
2. **Emotions** include feelings between unpleasant and pleasant, including neutral and mixed states. If $v(e)$ denotes the valence of emotion e , one simple ordering is:

$$-1 \leq v(e) \leq 1,$$

where -1 is unpleasant, 0 is neutral, and 1 is pleasant.

3. **Sensations** include the classical five: sight, sound, touch, smell, and taste.
4. **Perceptions** include recognizing or interpreting something as “pen,” “voice,” “danger,” or “my hand.”
5. **Emptiness** means that no content from one or more of these categories is represented in the selected description of the present moment.

These categories interact and frequently appear together. Anxiety, for example, may include a thought about returning to work, butterflies in the stomach, a sinking heart, an unpleasant feeling, and the perception of a future situation as threatening. The categories are labels used in observation to capture what appears in the mind, not sealed biological boxes. Figure 3 pictures their coexistence in one field, which is called an idea space and denoted by $\mathcal{A}$.

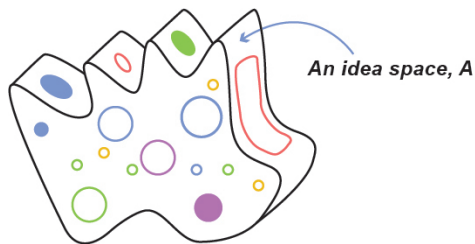


Figure 3: An idea space. The different marks represent thoughts, emotions, sensations, and perceptions that may appear, combine, and change within one field.

We can describe this more carefully. In the formal model, each basic content belongs to one category. Let J be the collection of people under discussion. For each person $j \in J$, begin with:

$$\mathcal{B}_j = T_j \sqcup E_j \sqcup S_j \sqcup P_j$$

where T_j, E_j, S_j , and P_j are the thoughts, emotions, sensations, and perceptions available in the model for that person. Simply put, $\mathcal{B}_j$ is the complete ingredient list of modeled contents available to person j .

A momentary state $a_j(t)$ is any selected subset of those contents for person j at one particular time t :

$$a_j(t) \subseteq \mathcal{B}_j.$$

The person's complete idea space, $\mathcal{A}_j$, is the collection of every possible momentary state:

$$\mathcal{A}_j = \mathcal{P}(\mathcal{B}_j).$$

The symbol $\mathcal{P}$ means “the power set,” or the set of all subsets of $\mathcal{B}_j$. Therefore $\mathcal{B}_j$ is the starting list of modeled contents, while $\mathcal{A}_j$ is the collection of all possible subsets, finite or infinite.

For clarity, $\mathcal{A}_j$ is an idealized space of possibilities, not a record of the states person j actually experiences across a lifetime. At any time t , person j experiences one momentary state $a_j(t) \in \mathcal{A}_j$.

For example, suppose a simplified $\mathcal{B}_j$ contains only the sound of a wave, warmth from the sun, and the thought “beautiful sunset.” One possible $a_j(t)$ contains only the first two; another contains all three; and $\mathcal{A}_j$ contains those states, every other combination, and the empty state.

Two people may have similar contents, but each presentation and experiential history belongs to the person experiencing it. Contents are indexed by the person experiencing them. Thus:

$$\mathcal{B}_j \cap \mathcal{B}_k = \emptyset \quad \text{for } j \neq k,$$

and two different people have distinct subject-indexed idea spaces:

$$j \neq k \implies \mathcal{A}_j \neq \mathcal{A}_k.$$

Emptiness is the fifth item in the practical description of an idea space. Mathematically, it appears whenever a selected category contains nothing. For example:

$$a_j(t) \cap T_j = \emptyset$$

means that no thought is present in the modeled state at time t . Imagine becoming completely absorbed in the sound and physical sensation of playing music, running, or watching a sunset. For a moment, the selected state may contain sensations and perceptions without a represented verbal thought. At the limiting case:

$$a_j(t) = \emptyset$$

represents no selected content from any of the four content categories. Emptiness is therefore not one more thought sitting beside the others; it is the possibility that one category, or the whole selected state, contains nothing.

2.1 Consciousness as light

Consciousness remains an enigma in our model. The best we can do is describe consciousness as the light one casts onto an idea space. The metaphor separates the full momentary state from the portion illuminated in awareness. If $a_j(t)$ is person j 's momentary state, let:

$$c_j(t) \subseteq a_j(t), \quad a_j(t) \in \mathcal{A}_j,$$

represent the contents of that state that are presently illuminated by consciousness. One may notice a thought, shift toward sensation, open to sound, or rest without a salient object.

This paper does not claim to explain what consciousness ultimately is or how it forms. It offers a way to describe and observe the mind's apparent contents.

2.2 The “I” inside the idea space

Your name or identity is an amalgamation of thoughts, emotions, sensations, perceptions, relationships, memories, and consciousness at a particular point in spacetime. It is always changing, or impermanent. Shunryu Suzuki writes:[25]

“What we call ‘I’ is just a swinging door which moves when we inhale and when we exhale.”

Return to Feynman's chair. If the boundary between chair, paint, dirt, and surrounding air is only an approximation, where do you start and where do you end? Your name—the ordinary “I”—is also an approximation used by others, and sometimes by yourself, to capture who you are at a particular event in spacetime. The name itself is another appearance within your idea space, alongside memory, body image, role, and ownership.

As Alan Watts says, “The true Self is Non-Self.”[27] Non-Self is not a detached void. It is the amalgamation of all the layers that participate in who you are: idea space, molecules, cells, body, name, relationships, culture, country, planet, and the wider universe. Because these layers are always changing and no final boundary separates one cleanly from the next, the “I” remains a useful approximation rather than one isolated object.

3 The empty set and zero measure

3.1 The empty set

Imagine a box containing three apples. Let F be the set of apples currently inside:

$$F = \{\text{apple}_1, \text{apple}_2, \text{apple}_3\}.$$

Remove the apples one by one. After the final apple is removed, the box still exists, but the set of apples inside it has no members:

$$F = \emptyset.$$

The symbol $\emptyset$ denotes the unique set with no elements. The collection has been defined and contains nothing.

The same notation applies to an idea space. If a moment contains sensations and perceptions but no thought, then $a_j(t) \cap T_j = \emptyset$. If the modeled state contains no selected content from any category, then $a_j(t) = \emptyset$. In this precise sense, an idea space contains the possibility of emptiness.

3.2 What measure zero means

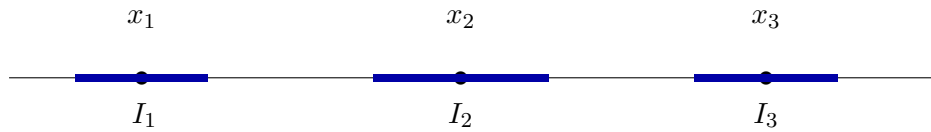
For a set $X \subseteq \mathbb{R}$, measure zero has a precise meaning:

A set X has Lebesgue measure zero if, for every number $\varepsilon > 0$, one can cover every point of X with intervals whose combined length is less than ε .

The intervals may be different sizes and may overlap. The only requirements are that together they cover X and that their total length can be made as small as we please. Formally, for every $\varepsilon > 0$, there are intervals $I_1, I_2, \dots$ such that:

$$X \subseteq \bigcup_{k=1}^{\infty} I_k \quad \text{and} \quad \sum_{k=1}^{\infty} |I_k| < \varepsilon.$$

Figure 4 makes one such covering visible. In this simple example, $X = \{x_1, x_2, x_3\}$. Each point x_i is covered by an interval I_i , and the combined length of the three intervals is smaller than the chosen number $\varepsilon > 0$. The intervals can be made smaller again for every smaller positive ε .



$$X \subseteq I_1 \cup I_2 \cup I_3, \quad |I_1| + |I_2| + |I_3| < \varepsilon, \quad \varepsilon > 0$$

Figure 4: Covering a set with arbitrarily short intervals. For any chosen $\varepsilon > 0$, the covering intervals can be selected so their total length is smaller than ε .

Lebesgue outer measure packages this test into one number. Consider every possible countable interval cover of X , calculate the total length of each cover, and determine the greatest lower bound of those totals. The symbol $\inf$, or infimum, denotes that greatest lower bound:

$$m^*(X) = \inf \left\{ \sum_{k=1}^{\infty} |I_k| : X \subseteq \bigcup_{k=1}^{\infty} I_k \right\}.$$

Thus the statement “ X has measure zero” can be written compactly as:

$$m^*(X) = 0.$$

Now, something fascinating is that a set containing elements can have zero measure, while the empty set also has zero measure. More formally, both have zero outer measure and look the same:

$$m^*({0}) = 0 = m^*(\emptyset).$$

But, clearly, the set containing an element and the empty set are different:

$${0} \neq \emptyset.$$

In other words,

$$\text{zero measure} \neq \text{nothing}.$$

Zero measure does not necessarily imply nothingness. Under this measurement, a nonempty zero-measure set and the empty set are indistinguishable to an observer whose only available output is that specified measure: both receive the value zero.

For instance, imagine opening two identical doors. Behind the first door is nothing. Behind the second is an uncountable collection with zero total measure. If the only reading available from the hallway is that specified measure, both doors return zero and look the same. Figure 5 illustrates the problem from the outside observer’s restricted measurement perspective.

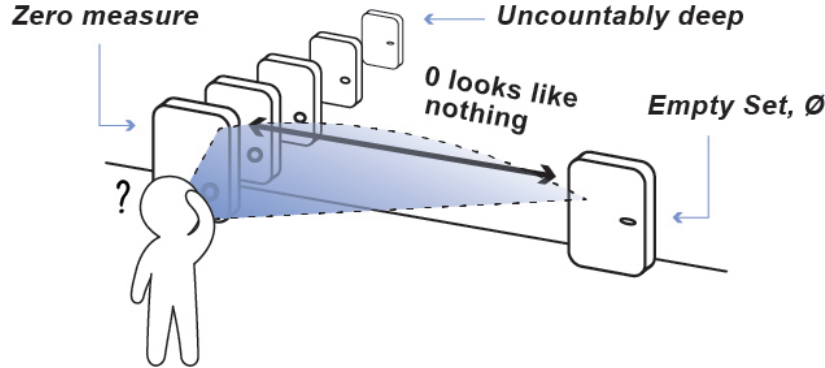


Figure 5: Zero measure looks like nothing under the specified measurement. One door hides uncountable zero-measure depth; the other hides nothing; both return zero and look the same through the measurement available from the hallway.

If the observer receives positional, structural, or membership information, however, the sets may be distinguishable. For example, a line segment has zero two-dimensional area but remains visible and locatable because our eyes receive light and structural information about it.

In our case, another person’s idea space does not appear before our eyes as a spatial object. We can observe the person’s body, words, behavior, and neural activity, but we do not directly receive

the contents constituting that person’s first-person experience. Under the outside-observer measure developed below, those private contents and an empty set of presented first-person contents both return zero and are indistinguishable by that measurement alone.

Simply put, what looks like nothing may contain uncountable depth. As Martin Rees said, [24] “Absence of evidence is not evidence of absence.”

Later on, the Cantor set will provide a rigorous, mathematical example of this construct. It is nonempty, uncountable, and has Lebesgue measure zero.

4 The idea space metric

A metric is a rule for assigning distance between points in a set. For all points x, y, z , a metric, or distance, d satisfies:

1. $d(x, y) \geq 0$, with $d(x, y) = 0$ if and only if $x = y$;
2. $d(x, y) = d(y, x)$;
3. $d(x, z) \leq d(x, y) + d(y, z)$.

A pseudometric requires nonnegativity, symmetry, and the triangle inequality, but it does not require $d(x, y) = 0$ to imply $x = y$. Distinct points may therefore have distance zero. To model the fact that an outside observer cannot directly see, occupy, or compare the states of another person’s mind, this paper chooses the following constant-zero pseudometric:

$$d(x, y) = 0 \quad \text{for every } x, y.$$

Figure 6 compares two metrics and one pseudometric. Euclidean distance is the familiar Pythagorean distance. In the discrete metric, every pair of distinct points is one unit apart. In the idea space pseudometric, every pair of states is zero units apart to an outside observer.

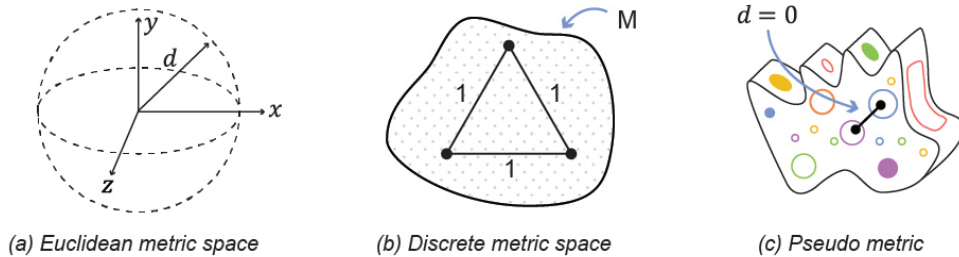


Figure 6: Two metrics and one pseudometric. (Left) Euclidean distance varies continuously; (Center) discrete distance places distinct points one unit apart; (Right) the idea space pseudometric places all modeled states at zero distance under the outside-observer rule.

For a person j and an outside observer k , where $k \neq j$, write the idea space metric more formally as:

$$d_{\text{IS}}^{(k|j)}(a_j, b_j) = 0 \quad \text{for every } a_j, b_j \in \mathcal{A}_j.$$

This means that any two states a_j and b_j in person j 's idea space appear to have zero distance from one another relative to an outside observer k . After fixing j and k , we suppress the indices for readability and write:

$$d_{\text{IS}}(a, b) = 0 \quad \text{for every } a, b \in \mathcal{A}.$$

In simple words, an outside observer cannot directly enter, see, or compare two states in another person's idea space. The pair is assigned distance zero under this outside-observer pseudometric, even though the states clearly remain different to the person experiencing them.¹

4.1 The private-image experiment

To better understand the idea space metric, let's consider the following experiment.

Hold something in your hand, like a pencil, a book, or a phone. Clearly, you can measure it. You can feel it. Others can see it.

Now close your eyes and bring a mental image of what you are holding into your mind. Can anyone else see the image in your head in the same direct way? No. There is not nothing there—clearly there is something there. It is directly presented only to you. This holds for your thoughts, emotions, sensations, and perceptions.

The pseudometric represents that difference. Distinct private states receive zero distance for the outside observer while remaining distinct to the person having them. Of course, a person can use speech and behavior to report an experience, while physiology and neural recordings can provide rich indirect evidence. The model does not claim that outsiders can learn nothing. Rather, until a first-person experience itself can be transferred, an outside observer cannot occupy, see, or experience another person's idea space.

4.2 From zero distance to zero external measure

This subsection connects the metric to the second proposed property of an idea space. A smooth spacetime is locally modeled with four real coordinates, commonly written (x, y, z, t) , but that does not make every object associated with spacetime a subset of the real line. An idea space is a state space associated with a person at events in spacetime; it is not itself a line or a region of spacetime. It therefore does not automatically inherit ordinary length, area, or volume from spacetime's real-valued coordinates.

The Lebesgue definition in the preceding section uses intervals on the real line. To measure an idea space, we instead need a notion of size built from its own pseudometric.

Hausdorff measure extends the interval-covering idea by measuring the diameters of covering sets.[10] Under the idea space pseudometric, every pairwise distance is zero, so every covering set also has diameter zero. Its total measured size is therefore zero. More precisely, every subset has zero

¹An earlier version described the idea space metric as a metric for general relativity. It is not a spacetime metric. The present paper treats it as a pseudometric for first-person experience and keeps ordinary spacetime geometry separate.

s -dimensional Hausdorff measure for $s > 0$.²

This is what the paper means by **zero external measure**: zero size under the outside-observer pseudometric. The zero Hausdorff measure is a consequence of this chosen observational structure.

4.3 Measurement depends on observer, rule, and scale

How can the same object look different at different scales?

In relativity, we see a person's proper time is directly affected by their motion, while fractal geometry teaches us to specify the scale and ruler. Two observers following different worldlines may accumulate different proper times; two measurements made at different resolutions may return different lengths.

For instance, the coastline of Britain shows why a measurement must name its ruler. A long measuring stick skips fine detail. A shorter stick follows more of the coastline and usually produces a greater total length, as Figure 7 illustrates.[18]

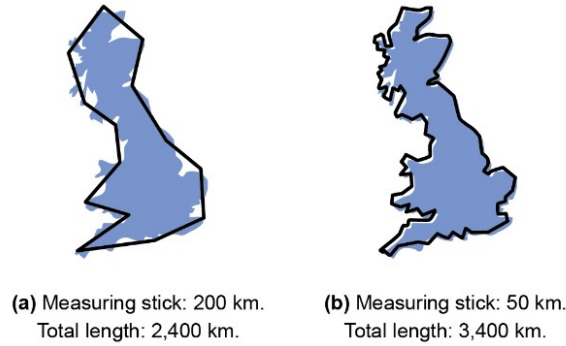


Figure 7: The coastline depends on measuring scale. Smaller measuring sticks capture more detail and can produce a larger measured total. The displayed 2,400 km and 3,400 km values are illustrative.

The coastline has not become arbitrary; its measured length depends on the ruler. Richardson's empirical data suggest a self-similarity dimension near $D = 1.25$ for the west coast of Great Britain. For a hypothetical normalized self-similar curve with initial scale $L_0 = 1$ meter, the scaling rule $L(\epsilon) = L_0(L_0/\epsilon)^{D-1}$, evaluated at the resolution $\epsilon = \ell_P$, where the Planck length $\ell_P \approx 1.6 \times 10^{-35}$ meters is a fundamental length scale, gives approximately 5×10^8 meters, or 500,000 km. This calculation describes a hypothetical normalized curve. Obviously, it is impossible to tell whether the physical coastline remains self-similar down to the Planck scale.

In the same way, speech, neural recordings, and first-person awareness reveal different aspects of the first-person experience. The idea space pseudometric asks one specific question: what direct first-person distance can an outside observer measure between another person's private states? In the model, that distance is zero. What looks like nothing from the outside may remain an immense landscape of experience to the person within it.

²Formally, $\text{diam}_{d_{\text{IS}}}(E) = 0$ and $\mathcal{H}_{d_{\text{IS}}}^s(E) = 0$ for every $E \subseteq \mathcal{A}$ and every $s > 0$. The restriction $s > 0$ matters; zero-dimensional Hausdorff measure requires separate treatment. Normalizing constants do not affect the zero-measure conclusion.

5 The topology of an idea space

A topology describes grouping, continuity, and neighborhoods without necessarily assigning a numerical distance.[20]

5.1 Open, closed, and clopen

Take the plane $M = \mathbb{R}^2$. The open unit disk:

$$O = \{(x, y) \in M : x^2 + y^2 < 1\}$$

excludes its circular boundary. Its complement:

$$O^c = M \setminus O = \{(x, y) \in M : x^2 + y^2 \geq 1\}$$

includes that boundary and is closed. Conversely, the closed unit disk $C = \{(x, y) \in M : x^2 + y^2 \leq 1\}$ includes its boundary, while its complement $C^c = M \setminus C = \{(x, y) \in M : x^2 + y^2 > 1\}$ is open. Thus the complement of an open set is closed, and the complement of a closed set is open.

Figure 8 shows both relationships: including the circular boundary produces the closed disk, while excluding it produces the open disk.

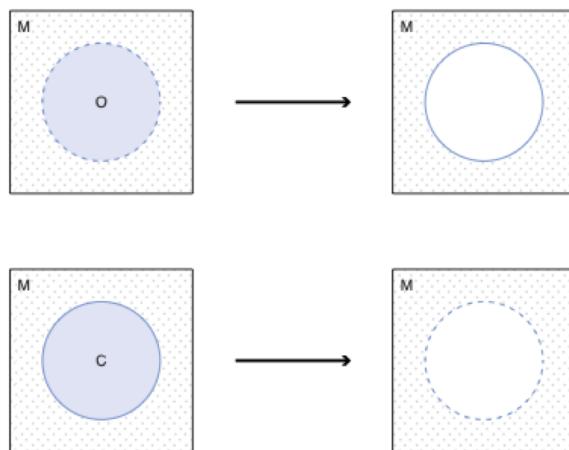


Figure 8: Open and closed disks and their complements. (Top) The complement of the open disk O includes its circular boundary and is closed. (Bottom) The complement of the closed disk C excludes that boundary and is open.

A set that is both open and closed is called **clopen**. Now consider the whole space M . It contains all of its limit points, so it is closed. Its complement is the empty set:

$$M^c = \emptyset.$$

The empty set has no missing limit points, so it is also closed. Because the complement of a closed set is open, M and $\emptyset$ are both open as well. Therefore:

$$\boxed{M \text{ and } \emptyset \text{ are both clopen.}}$$

5.2 Why an idea space is topological and clopen

A topological space is a set M together with a collection τ of subsets designated as open. The collection must satisfy three conditions:

1. $\emptyset \in \tau$ and $M \in \tau$;
2. the union of any collection of members of τ remains in τ ;
3. the intersection of finitely many members of τ remains in τ .

Now return to person j 's idea space $\mathcal{A}_j$. The individual state $a_j(t)$ is one point, or element, of $\mathcal{A}_j$. An open set $U \subseteq \mathcal{A}_j$ is a collection of possible states, not one basic thought or sensation from $\mathcal{B}_j$. A particular singleton $\{a_j(t)\}$ can be declared open only if the chosen topology contains it.

Two topologies represent the two perspectives developed in this paper. From the outside-observer perspective, the zero pseudometric induces the minimal, indiscrete topology:

$$\tau_j^{\text{out}} = \{\emptyset, \mathcal{A}_j\}.$$

Only the empty set and the complete idea space are open. Like our example above, their complements are also open, so both are clopen. Under this topology, distinct private states cannot be separated through the outside-observer structure.

From the first-person perspective, choose the discrete topology:

$$\tau_j^{\text{in}} = \mathcal{P}(\mathcal{A}_j).$$

In a discrete topology, every subset of $\mathcal{A}_j$ is open. Because the complement of each subset is also open, every subset is closed as well. For example, if a toy idea space is $\mathcal{A}_j = \{a, b\}$, then $\{a\}$ is open and its complement $\{b\}$ is also open; therefore $\{a\}$ is both open and closed. In particular, each momentary state, represented topologically by the singleton

$$\{a_j(t)\} \subseteq \mathcal{A}_j,$$

is clopen from the first-person perspective.

These two topologies are different structures on the same underlying state space, representing different forms of access. The outside topology is induced by the zero pseudometric; the inside topology is an additional modeling choice that preserves the topological distinctness of each lived state.

The complete idea space $\mathcal{A}_j$ and the empty set are clopen under both.

6 The Cantor set, uncountability, and topological singularity

6.1 Numbers, functions, and counting

Counting is more than pointing at objects. A set is **countably infinite** when its members can be paired, without omissions or duplicates, with the natural numbers:

$$\mathbb{N} = \{1, 2, 3, \dots\}.$$

Integers, denoted by the symbol $\mathbb{Z}$, include all negative whole numbers, zero, and positive whole numbers. Rational numbers, denoted by $\mathbb{Q}$, are numbers of the form p/q , where $p, q \in \mathbb{Z}$ and $q \neq 0$. Irrational numbers are numbers that cannot be written in the form p/q , such as $\sqrt{2}$ or π , and their decimal expansions neither terminate nor become eventually periodic. Real numbers, denoted by $\mathbb{R}$, include both rational and irrational numbers and make up the entire real number line.

A function maps each input to an output. The function is injective if distinct inputs have distinct outputs, surjective if every target output is reached, and bijective if the function is both injective and surjective. Any set that has a bijection with $\mathbb{N}$ is countably infinite.[13]

Figure 9 makes that pairing concrete. Each orange receives exactly one natural number, and each number used reaches exactly one orange.

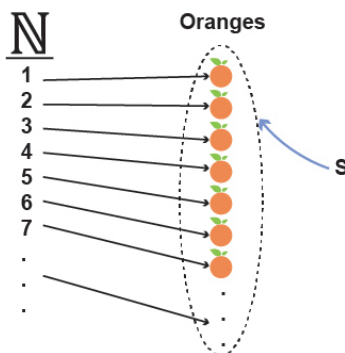


Figure 9: What it means to count. Each orange is paired with one natural number. Even an infinite collection is countable if such a list reaches every member.

A set can be finite or countably infinite—both are called **countable**. The natural numbers, integers, and rational numbers are all countable even though each is infinite. They have the same size of infinity because each can be placed in a complete list indexed by $\mathbb{N}$. Infinity therefore comes in different sizes. The real numbers and the Cantor set are a larger size of infinity and are **uncountable**: no list indexed by the natural numbers can contain every member.

6.2 The Cantor set has zero measure

The Cantor set provides an example of a mathematical object that is uncountable and has zero Lebesgue measure.

Start with:

$$C_0 = [0, 1].$$

Remove its open middle third, $(1/3, 2/3)$, leaving:

$$C_1 = [0, 1/3] \cup [2/3, 1].$$

Remove the open middle third of each remaining interval and repeat indefinitely. At stage n , there are 2^n intervals, each of length 3^{-n} . Their total length is:

$$m(C_n) = 2^n 3^{-n} = \left(\frac{2}{3}\right)^n.$$

The Cantor set is the intersection of every stage:

$$C = \bigcap_{n=0}^{\infty} C_n.$$

Because:

$$\lim_{n \rightarrow \infty} \left(\frac{2}{3}\right)^n = 0,$$

the Cantor set has Lebesgue measure zero.

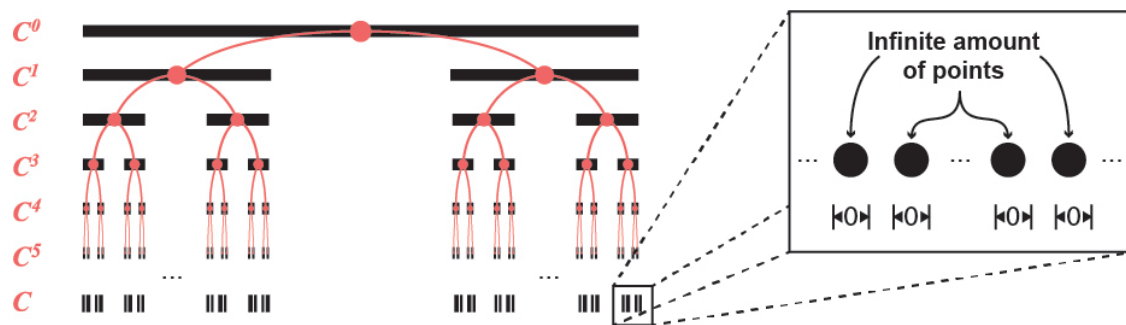


Figure 10: The middle-thirds Cantor set. The limiting object contains points rather than intervals, while the total remaining length approaches zero as $(2/3)^n \rightarrow 0$.

6.3 The Cantor set is uncountable

At each stage, label the left surviving interval 0 and the right surviving interval 2. Each point in the Cantor set receives an infinite address made only of zeros and twos, as Figure 11 shows.

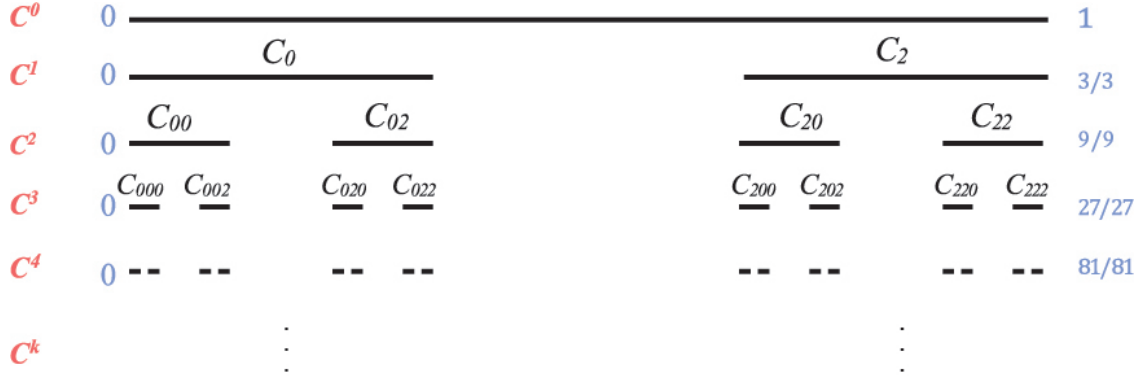


Figure 11: Addresses in the Cantor set. Each finite stage fixes a longer left/right prefix. At the limit, an infinite address identifies a Cantor point.

For example, $C_{002}\dots$ means: go left, then left, then right, and continue according to the remaining digits. At the limit, every Cantor point has an address with infinitely many positions. The notation:

$$\{0, 2\}^{\mathbb{N}}$$

means exactly this: the set of all infinite addresses made from 0 and 2, one digit for each natural-numbered position.

Now suppose the Cantor set were countable. Then all of its addresses could be placed in a list, with one address beside each natural number, as Figure 12 proposes.

$\mathbb{N}$		Cantor Set
1	→	$C\omega_{11}\omega_{12}\omega_{13}\omega_{14}\omega_{15}\omega_{16}\omega_{17}\dots$
2	→	$C\omega_{21}\omega_{22}\omega_{23}\omega_{24}\omega_{25}\omega_{26}\omega_{27}\dots$
3	→	$C\omega_{31}\omega_{32}\omega_{33}\omega_{34}\omega_{35}\omega_{36}\omega_{37}\dots$
4	→	$C\omega_{41}\omega_{42}\omega_{43}\omega_{44}\omega_{45}\omega_{46}\omega_{47}\dots$
5	→	$C\omega_{51}\omega_{52}\omega_{53}\omega_{54}\omega_{55}\omega_{56}\omega_{57}\dots$
6	→	$C\omega_{61}\omega_{62}\omega_{63}\omega_{64}\omega_{65}\omega_{66}\omega_{67}\dots$
7	→	$C\omega_{71}\omega_{72}\omega_{73}\omega_{74}\omega_{75}\omega_{76}\omega_{77}\dots$
$\vdots$	→	$\vdots$

Figure 12: A proposed count of all Cantor addresses. If the Cantor set were countable, every infinite left/right address would appear somewhere on this list.

Read the diagonal digits: the first digit of the first address (ω_{11}), the second digit of the second address (ω_{22}), the third digit of the third address (ω_{33}), and so on. Construct a new address $a = C_{a_1a_2a_3\dots}$ by switching each diagonal digit:

$$a_i = \begin{cases} 2, & \omega_{ii} = 0, \\ 0, & \omega_{ii} = 2. \end{cases}$$

The new address differs from the first listed address in its first position, the second address in its second position, and every i -th address in its i -th position. It cannot appear anywhere in the supposedly complete list. Figure 13 makes the missing address visible.

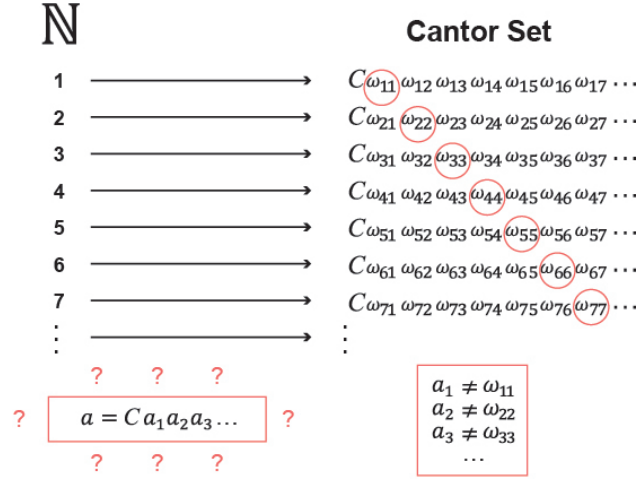


Figure 13: The missing diagonal address. Switching every highlighted diagonal digit creates a valid Cantor address that differs from every address on the list.

The assumption that the Cantor set could be counted therefore fails. The Cantor set is uncountable.

6.4 The modeled idea space is uncountable

The binary-address architecture used for the Cantor set can now be applied to the idealized field of possible mind states. This correspondence concerns combinations and cardinality. The idea space's zero external measure follows separately from the outside-observer pseudometric.

Return to person j 's basic contents. For this proof, make the conservative modeling assumption that $\mathcal{B}_j$ is countably infinite:

$$\mathcal{B}_j = \{\beta_{j1}, \beta_{j2}, \beta_{j3}, \dots\}.$$

Here, β_{jn} means the n -th basic thought, emotion, sensation, or perception in person j 's model.

If $\mathcal{B}_j$ were already uncountable, then $\mathcal{A}_j = \mathcal{P}(\mathcal{B}_j)$ would be uncountable immediately, since each $\beta \in \mathcal{B}_j$ determines a distinct singleton state $\{\beta\} \in \mathcal{A}_j$.

We assume $\mathcal{B}_j$ is countably infinite because the visual, auditory, and bodily fields admit indefinite refinement across position, frequency, intensity, and other dimensions of experience. The depth of our physical world is commonly described by real-valued coordinates, which form an infinite continuum. Ordinary, nonempty lived states may therefore contain infinitely many modeled distinctions, while the empty state remains a formal possibility.

With this countably infinite ingredient list in place, taking the full power set $\mathcal{A}_j = \mathcal{P}(\mathcal{B}_j)$ permits every complete yes-or-no assignment across this sequence, including both finite and infinite combinations. The proof below establishes that this complete idealized state space is uncountable.

A finite example shows how combinations multiply. If:

$$\mathcal{B}_j = \{\text{wave sound, warmth, "beautiful sunset"}\},$$

then $\mathcal{A}_j$ contains $2^3 = 8$ possible states: the empty state, three one-content states, three two-content states, and the state containing all three. Every new basic content doubles the number of possible combinations.

Because $\mathcal{B}_j$ is countably infinite, its basic contents can be numbered by the natural numbers. Let $e : \mathbb{N} \rightarrow \mathcal{B}_j$ record that numbering, so $e(1) = \beta_{j1}$, $e(2) = \beta_{j2}$, and in general $e(n) = \beta_{jn}$.

We can now describe any possible state by asking one yes-or-no question at every numbered position: is $e(n)$ present in this state? Write 1 when it is present and 0 when it is absent. Define:

$$\Psi(a)_n = \begin{cases} 1, & e(n) \in a, \\ 0, & e(n) \notin a. \end{cases}$$

Every possible state $a \in \mathcal{A}_j$ therefore receives one infinite binary address $\Psi(a)$. The map Ψ is a bijection: each state determines exactly one address, and each address $u \in \{0, 1\}^{\mathbb{N}}$ reconstructs exactly one state, $\{e(n) : u_n = 1\}$. For example:

basic content	β_{j1}	β_{j2}	β_{j3}	β_{j4}	β_{j5}	$\cdots$
state a	1	0	1	0	1	$\cdots$

For a concrete example, suppose the first five basic contents are:

β_{j1} = the sound of waves, β_{j2} = warmth from the sun, β_{j3} = the thought "beautiful sunset,"
 β_{j4} = the smell of salt water, β_{j5} = a feeling of calm.

The prefix 10101... describes a state containing the sound of waves, the thought "beautiful sunset," and the feeling of calm, while not containing warmth or the smell of salt water. The remaining digits continue the same yes-or-no record for every other basic content. The notation:

$$\{0, 1\}^{\mathbb{N}}$$

means the set of all such infinite yes-or-no addresses. It is the binary version of the Cantor set's infinite left-or-right addresses.

Now make an assumption that the proof will disprove: suppose $\mathcal{A}_j$ were countable, so every possible state could be listed:

$$s_1, s_2, s_3, \dots$$

Construct a new state s_{Δ} by reversing the diagonal membership decision:

$$s_{\Delta} = \{\beta_{jn} \in \mathcal{B}_j : \beta_{jn} \notin s_n\}.$$

The new state differs from s_1 on β_{j1} , from s_2 on β_{j2} , and from every s_n on β_{jn} . For instance, if the first listed state s_1 contains the sound of waves, s_{Δ} does not. If the second listed state s_2 does not contain warmth, s_{Δ} does. Continuing this rule ensures that s_{Δ} differs from every listed state in at least one basic content. Therefore s_{Δ} cannot appear anywhere on the proposed complete list. This contradiction proves that $\mathcal{A}_j$ is uncountable.

Figure 14 visualizes the central analogy: the Cantor set and the modeled idea space both combine zero measure, under their respective specified measures, with uncountable depth. In the modeling spirit described in the introduction, a mathematical object helps describe the proposed properties of a physical mind.

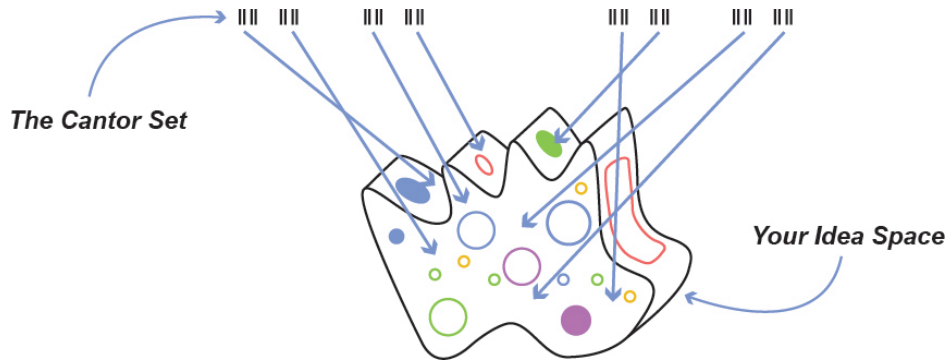


Figure 14: Mapping the Cantor set to an idea space. Both combine zero measure, under their respective specified measures, with uncountable depth. Infinite Cantor-style addresses map to distinct possible idea-space states.

The old thought experiment gives this proof an intuitive shape. Picture yourself in 1680, trying to list every idea available in the world. You could write indefinitely and still fail to include Newton’s 1687 mathematical formulation of gravity, later physical theories, or the idea space itself. Newton did not create the physical attraction between bodies; he introduced a formulation that changed the landscape of ideas available to later minds.

Lived experience is impermanent: moment by moment, thoughts, emotions, sensations, and perceptions arise, combine, and pass away. In the model, this change is represented by $a_j(t)$, the state experienced at time t , moving through the uncountable field of possibilities $\mathcal{A}_j$.

6.5 Topological singularity

To classify these phenomena under a larger umbrella, this paper introduces the term **topological singularity**:

A topological singularity is a modeled object that is uncountable and has measure zero under a specified measure.

The term names the combined cardinal-and-measure classification defined in this paper. It is coined terminology specific to this framework rather than standard nomenclature for topological or spacetime

singularities. The Cantor set qualifies because it is uncountable and has Lebesgue measure zero. The modeled idea space $\mathcal{A}$ qualifies because its complete state space is uncountable and it has zero measure under the outside-observer measure.

A possible cosmological extension concerns the comoving region R that develops into our present observable universe. In a classical FLRW model, the proper spatial volume of a fixed comoving region scales with the cube of the scale factor:[6]

$$V_R(t) = \alpha(t)^3 V_R^{\text{comoving}}.$$

At every regular time $t > 0$, this region has positive volume. In the classical extrapolation toward the Big Bang, however, $\alpha(t) \rightarrow 0$, and therefore $V_R(t) \rightarrow 0$. The comoving region R is modeled using real-valued spatial coordinates, so its continuum of coordinate labels is uncountable. If its limiting volume is defined by:

$$V_R(0) := \lim_{t \rightarrow 0^+} V_R(t) = 0,$$

then the model pairs an uncountable set of comoving labels with zero limiting spatial volume. Under the terminology introduced here, this formal limiting representation of our observable region can be treated as a topological singularity.

This is a limiting statement about the region R , not about the Big Bang as a whole. Because $t = 0$ is not a regular spatial slice of classical spacetime and general relativity is expected to become incomplete near this limit, the statement refers only to $t \rightarrow 0^+$. It is a formal analogy based on limiting volume rather than a construction of a physical region at $t = 0$.

7 The Singularity Sunset

The Sunset Conjecture shows how your idea space fits within the universe around you. As stated in the introduction, it is two-fold:

- (a) every observer lives at the center of their own observable universe where everything they see is in the past; and
- (b) their idea space, with uncountable possibility and zero external measure, is associated with the observer at that central observer-event.

The sunset provides the everyday version. Two people share one sun, one ocean, and one physical world, while each receives a different reflected path. The same structure extends outward: nearby light, distant galaxies, and evidence of the early universe all reach a particular observer along a particular causal history.

7.1 At the center of your observable universe

An observer is at the center of their *observable* universe by construction. The phrase does not mean that the observer occupies the physical center of the entire universe. It means that the observable

region is defined by the signals that can reach that observer here and now. On a constant-time slice in a standard nearly isotropic FLRW description, this region is approximately spherical around the observer.

Everything we see is in the past because light takes time to travel, even at 186,000 miles per second. We see the Sun as it was about eight minutes ago, because it took eight minutes for that light to travel 93 million miles. For those same reasons, we see the center of the Milky Way as it was roughly 26,000 years ago and distant galaxies appear millions or billions of years in the past. Figure 15 illustrates this lookback structure. Its nested boundaries represent progressively older information arriving at the observer, not literal shells or a privileged cosmic center.

This raises the natural question: if every visual observation looks backward in time, what is the farthest back we can see?

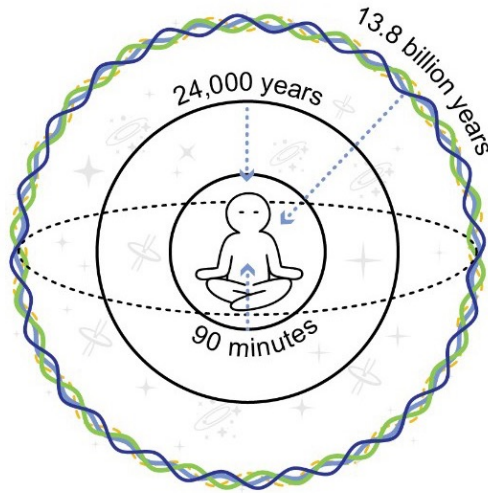


Figure 15: An observer at the center of an observable universe. Light and causal information from different lookback times arrive at one observer. The rings are a schematic illustration of lookback depth, not physical walls around the universe, and are not to scale.

The oldest electromagnetic light directly observed is the cosmic microwave background, emitted from a finite-thickness last-scattering region about 380,000 years after the hot Big Bang.[22] It is not a photograph of the Big Bang itself. Earlier conditions are inferred from physical models, primordial element abundances, CMB patterns, and later cosmic structure. In standard cosmology, extrapolating still farther back leads toward the shared early-time boundary called the Big Bang.

Different observers therefore receive CMB photons and other evidence of the same early universe along different paths ending at their own observer-events.

7.2 Events and past light cones

An **event** is one point in spacetime. In a chosen coordinate system it can be written $p = (x, y, z, t)$: three spatial coordinates and one time coordinate. Every observation occurs at such an event. The causal past of p is written:

$$J^-(p).$$

The set $J^-(p)$ contains every earlier event capable of influencing p . Its boundary, $\partial J^-(p)$, is the past light cone. Light approaching p follows this lightlike boundary; slower causal influences travel through the timelike interior, denoted $I^-(p)$. Thus $J^-(p)$ includes both the past light-cone boundary and its timelike interior. Events outside $J^-(p)$ cannot influence p ; they include events spacelike-separated from p as well as events in its causal future.[6] This distinction is clarified in Figure 16.

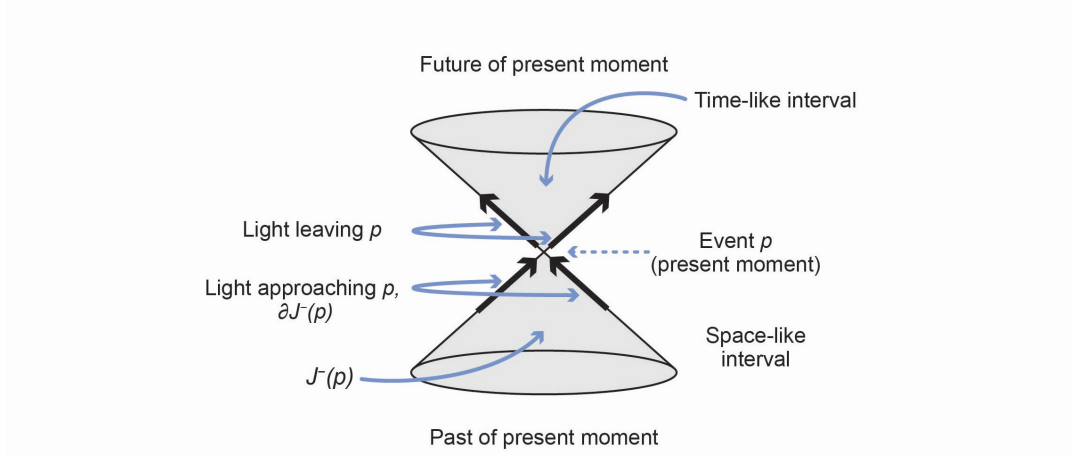


Figure 16: The causal structure around an event. Light approaching p travels on the past light-cone boundary; light leaving p travels on the future boundary. Below p , the past-cone interior is $I^-(p)$, its boundary is $\partial J^-(p)$, and the region outside the past and future cones is spacelike.

7.3 Where do you start, and where do you end?

Now, let's return to Feynman's chair. If an ordinary object cannot be given one final microscopic boundary, where does a person start and end? Think of the Self as layers of an onion. Moving inward, one finds the name, body, cells, molecules, and idea space. Moving outward, one finds relationships, culture, country, planet, galaxy, and ultimately the observer's entire observable universe.

idea space $\longleftrightarrow$ molecules $\longleftrightarrow$ cells $\longleftrightarrow$ person $\longleftrightarrow$ name $\longleftrightarrow$ country $\longleftrightarrow$ planet $\longleftrightarrow$ observable universe

The widest layer does not mean that the body physically fills the observable universe. It means that no account of the present person is independent of the causal history that reaches them from that universe. The body's matter has a cosmic history; language and memory come through relationships; visual experience is assembled from light emitted in the past.

Feynman's observation about physical boundaries does not by itself prove Non-Self; it motivates the philosophical interpretation developed here. In that interpretation, no single layer supplies a final boundary for what you are. The current "I"—your name—is the practical approximation for this entire changing structure at one particular event in spacetime.

7.4 The Singularity Sunset

The Singularity Sunset brings these two parts together. In the idealized causal picture, light reaching an event p travels along null paths traced by that event's past light cone. Moonlight, sunlight reflected across the ocean, light from the center of the Milky Way, and CMB photons all meet the observer after different travel times. In this sense, the past light cone is continually bringing earlier information to the observer.

For two distinct observer-events p_A and p_B , the causal pasts $J^-(p_A)$ and $J^-(p_B)$ usually overlap but do not have identical outer boundaries. The observers share one physical and cosmic history while receiving different paths through it.

The outer reach of an observable universe is therefore observer-relative. Direct vision ends at the last-scattering surface, while the physical model extrapolates toward the shared Big Bang boundary. Neither observer owns a separate Big Bang. Rather, electromagnetic signals and gravitational imprints from the early universe are received and measured from each observer's particular position in spacetime and along that observer's causal history. The Big Bang is shared; its effects are encountered from an observer-relative perspective, just as one sunset sends a distinct golden path to each viewer.

Figure 17 places the everyday sunset beside two overlapping causal pasts.

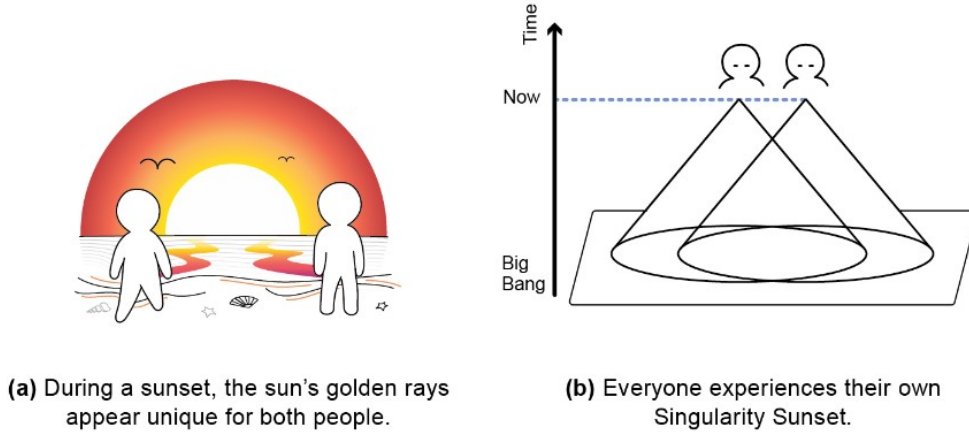


Figure 17: The Singularity Sunset. Two observers receive one shared cosmic history from distinct observer-events on a schematic equal-time slice. The plane labeled “Big Bang” is schematic; mathematically, the discussion concerns regular early-universe slices $t = \varepsilon$ and the limiting behavior as $\varepsilon \rightarrow 0^+$.

Associated with the observer at the center of this observable region is person j 's idea space $\mathcal{A}_j$, with uncountable possibility and zero external measure. Its contents are not directly presented to outside observers. Figure 18 returns that private space to the observer-event in the cosmic picture.

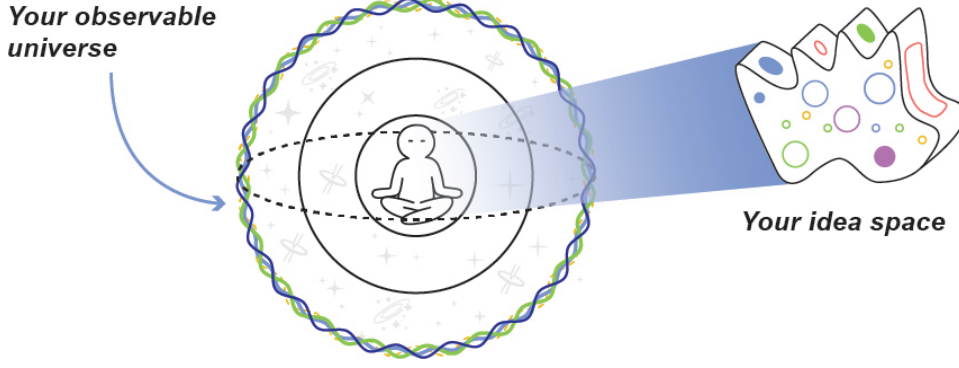


Figure 18: The idea space associated with the central observer-event. The observer’s causal perspective opens onto a private idea space that is not directly presented to outside observers.

Your Singularity Sunset is the whole layered relation: an idea space, a body and history, the sunset arriving now, the Milky Way seen in the past, and an observer-relative outer boundary reaching toward the beginning of the observable universe. The universe is shared; distinct observer-events generally have nonidentical causal boundaries and perspectives, while their first-person histories remain distinct.

8 Discussion: Where else could an idea space exist?

The idea space began with a human mind, but its vocabulary raises a broader question.

8.1 Personal and macro idea spaces

A **personal idea space**, $\mathcal{A}_j$, is person j ’s private first-person field. Other people cannot directly occupy it; they encounter expressions of it through words, images, music, mathematics, behavior, and other public signals. Communication creates correspondences between personal idea spaces as one person expresses an idea and another reconstructs it through a different history and perspective. The subject-indexed fields remain distinct while public signals create a shared layer through which they can coordinate.

Let Σ denote the collection of publicly available expressions, artifacts, and practices. A transmission from person j to person k can be pictured as:

$$a_j(t) \xrightarrow{\text{Enc}_j} s \xrightarrow{\text{Dec}_k} \hat{a}_k(t'), \quad s \in \Sigma,$$

where Enc_j encodes part of j ’s momentary state as a public signal s , and Dec_k decodes that signal into a related state in k ’s idea space. The reconstructed state $\hat{a}_k(t')$ need not reproduce the sender’s state exactly.

When s evokes an unusually faithful reconstruction of $a_j(t)$ in person k , it functions as a **koan** for k at time t' . The same signal may function as a koan for one person at a particular moment and not for another. This relation can be written:

$$\text{Dec}_k(\text{Enc}_j(a_j(t))) \sim_{jk} a_j(t),$$

where $\sim_{jk}$ means that, through reports, behavior, or other public criteria available to both people, the reconstruction is judged to preserve the sender’s intended idea with unusual fidelity. It does not mean that the two private states are literally identical or directly compared. Whether the signal functions as a koan still depends on the receiver and the moment.

When public expressions and signals are repeatedly transmitted, recognized, and sustained among a group G , the resulting pattern of correspondence and coordination forms a **macro idea space**, denoted $\mathcal{M}_G$. Public signals in Σ carry and renew this shared structure, but the macro idea space is the emergent shared reality rather than merely a collection of signals.

Personal idea spaces belong to individuals; macro idea spaces coordinate multiple individuals. Languages, currencies, laws, cultures, governments, companies, religions, and scientific frameworks are examples. At sufficient scale, these shared structures form **imagined realities**: realities made possible by collective agreement and participation. “Imagined” does not mean unreal or inconsequential; money, laws, and institutions can organize physical behavior precisely because people continue to sustain them. These imagined realities coexist with **objective realities**—features of the physical world, such as mountains and rivers, whose existence does not depend on collective agreement.[8]

Figure 19 illustrates how repeated communication coordinates distinct personal idea spaces into a sustained shared layer.

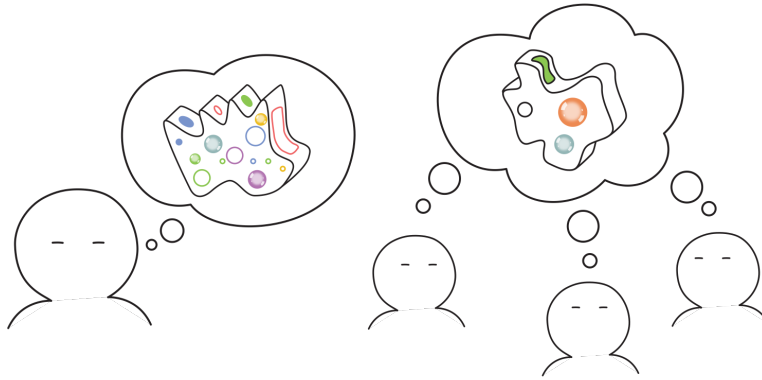


Figure 19: Personal and macro idea spaces. (Left) A personal idea space belongs to one individual. (Right) Repeated communication creates sustained correspondence and coordination among personal idea spaces, forming an emergent shared reality without erasing their first-person differences.

8.2 When was the first idea space?

Despite our unique experiences and backgrounds, there is an intrinsic sameness in all of us: a connection formed through shared evolutionary history. Even if one idea space differs from the next, our collective journey through time reveals a common inheritance. To be human means that there will always be at least one thing we have in common.[7]

That journey raises the larger question: *when was the first idea space?* Possible landmarks span enormous stretches of time:

- the earliest evidence of life, roughly 3.4 to 3.8 billion years ago;
- the emergence of eukaryotic cells, roughly 1.8 to 2.7 billion years ago;
- the earliest fossil evidence of sexual reproduction, roughly 1 billion years ago;
- the rise of early animals, roughly 600 to 800 million years ago;
- the divergence of the human and chimpanzee lineages, roughly 5 to 7 million years ago; and
- the earliest known *Homo sapiens*, roughly 300,000 years ago.

These dates are approximate and remain subject to revision as fossil evidence and dating methods improve.[3, 4, 16, 14]

Each landmark offers a different candidate. Did the first idea space begin with life itself, with a cell capable of integrating information, with an animal capable of sensation, or with a later form of self-awareness? Dogs and cats display perception, learning, memory, pain behavior, preference, and goal-directed action.[2] Plants detect signals, regulate themselves, and adapt. In one study, *Arabidopsis* exposed to vibrations produced by caterpillar chewing later increased chemical defenses and discriminated those vibrations from controls.[1]

These observations do not prove first-person experience, but within this framework it remains quite possible that dogs, cats, plants, and other living creatures have forms of idea space suited to their own ways of sensing and responding. Dinosaurs may likewise have had idea spaces shaped by their own bodies, environments, and histories.

Do individual cells have primitive idea spaces? Do stars have some radically different form of idea space because they are born, transform matter, and die? These questions become inseparable from consciousness. Can an idea space exist without consciousness? Does consciousness necessarily imply an idea space? Could an idea space have developed first as a mechanism for interpreting the environment, with consciousness emerging later to illuminate that field through one perspective?

The boundary remains nebulous because responsiveness, life, representation, and consciousness may not begin at the same moment. The framework cannot settle these questions in advance; its purpose is to make them precise: when does responsiveness become representation, when does representation become first-person experience, and at what point—if any—does consciousness mark the arrival of an idea space?

8.3 Abstracting large language models through an idea space

A large language model is, at one level, a large mathematical structure trained to predict the next token. Yet deployed systems increasingly combine language with images, audio, video, memory, tools, and robotics. The idea space lets us step back from a particular implementation and ask a simpler question: can the same four content categories and the possibility of emptiness used to describe a human mind provide a useful vocabulary for describing an artificial system?

Dogs and cats appear to have thoughts, emotions, sensations, and perceptions. They can also be at rest, with no obvious represented content. For language models, the comparison is less complete but

increasingly interesting:

- **Thoughts:** internal representations, intermediate reasoning, planning, and generated language can be described as functional analogues of thought.
- **Perceptions:** multimodal systems can recognize and transform language, images, audio, and video.
- **Sensations:** cameras and microphones provide analogues of sight and hearing. Robotic sensors increasingly provide touch. Smell and taste remain less developed.
- **Emotions:** this is harder. Human emotions often combine a thought, a bodily sensation, and a label: anxiety may include thinking about Monday morning, feeling a sinking heart or butterflies in the stomach, and naming the combination “anxiety.” A language model can represent and express emotional patterns, but whether anything is felt remains unknown.
- **Emptiness:** when a model is not processing a request, it may occupy an idle or inactive computational state. Whether that is meaningfully comparable to experienced emptiness depends on what kind of persistent system the model is part of.

This abstraction does not settle whether a language model is conscious. It asks whether idea space terminology can help us compare artificial systems as they acquire more of the capacities used to describe animal and human minds.[5] The value of the framework may lie in making the comparison explicit: which elements appear functionally present, which remain absent, and which cannot yet be determined by an outside observer?

Whether such a model could become “sentient” or “alive” depends on the unresolved question from the previous section: at what stage does consciousness enter the picture? If an idea space can exist before consciousness, an artificial system might satisfy the model’s structural description without feeling anything. If consciousness is inseparable from an idea space, the same evidence would carry a different implication. The framework exposes that fork by leaving the open question: Could organized silicon eventually support a genuinely different form of life or mind?[17]

Marshall McLuhan described technological co-evolution this way:[19]

“Man becomes, as it were, the sex organs of the machine world, as the bee of the plant world, enabling it to fecundate and to evolve ever new forms.”

9 Conclusion

An idea space applies mathematical language to observations of the mind. It tracks thoughts, emotions, sensations, and perceptions together with the possibility of an empty category or state. Its three modeled properties are uniqueness to the subject, zero external measure, and uncountable possibility. Mindfulness supplies the first-person observation; the pseudometric formalizes the specified outside-observer measurement; the outside and inside topologies distinguish two forms of access while keeping the whole space clopen; and the Cantor correspondence establishes the cardinality of the idealized possibility space. This paper calls a modeled object with uncountable cardinality and zero measure under a specified measure a topological singularity.

The Singularity Sunset returns the private field to a shared universe. Each observer receives light and causal influence at a particular event, while each first-person history is presented to one subject. This does not divide reality into private universes. It places distinct perspectives inside one common physical history. The same language may help us ask careful questions about animal, plant, collective, and artificial systems, but it does not answer in advance which of them are conscious or alive.

Do not take the model at face value. Test what it clarifies. Reject what fails. Build your own practice.

We share one cosmos, each from a unique perspective.

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